Shape and Topology optimization of frame structure by using force density method

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Abstract A new method for optimization of shape and topology of frame structure is presented in this paper. The frame structure is discretized by the Euler-Bernoulli element and each element is assumed to have circular cross section, hence the area and inertial of cross section can be determined by its radius or diameter. In order to find more reasonable shape and alleviate the difficulties caused by overlapping nodes, force density method is introduced to determine the nodal coordinates. By using the force density method, the nodal coordinate of the structure is an implicit function of force density and the corresponding connection matrix. Therefore, unlike the traditional optimization method, which usually takes the location of nodes and the cross section area as design variables, in this study the force density and diameter of beam element are considered to be design variables. Minimization of structural compliance under single or multiple loads is formulated as the optimization problem, with related volume constraint and rational bounds, namely

$$\begin{aligned} &\textit{Minimize } C\left(\boldsymbol{x}(\boldsymbol{q}),\boldsymbol{y}(\boldsymbol{q}),\boldsymbol{d}\right)=\boldsymbol{U}^{T}\left(\boldsymbol{x}(\boldsymbol{q}),\boldsymbol{y}(\boldsymbol{q}),\boldsymbol{d}\right)\boldsymbol{K}\left(\boldsymbol{x}(\boldsymbol{q}),\boldsymbol{y}(\boldsymbol{q}),\boldsymbol{d}\right)\boldsymbol{U}\left(\boldsymbol{x}(\boldsymbol{q}),\boldsymbol{y}(\boldsymbol{q}),\boldsymbol{d}\right) \\ &\textit{Subject to } \boldsymbol{q}_{lower}\le\boldsymbol{q}\le\boldsymbol{q}_{upper},\ \boldsymbol{d}_{lower}\le\boldsymbol{d}\le\boldsymbol{d}_{upper},\ \boldsymbol{x}_{lower}\le\boldsymbol{x}\le\boldsymbol{x}_{upper},\ \boldsymbol{y}_{lower}\le\boldsymbol{y}\le\boldsymbol{y}_{upper} \\ &\qquad V\left(x(\boldsymbol{q}),y(\boldsymbol{q}),\boldsymbol{d}\right)\le V_{initial} \end{aligned} \tag{1}$$

where $\boldsymbol{x}=(x_1, x_2,\ldots, x_n)$ and $\boldsymbol{y}=(y_1, y_2,\ldots, y_n)$ are the vector of nodal coordinate of nodes in *x*-*y* plane, and *n* represents the number of nodes in frame structrue; $\boldsymbol{q}=(q_1, q_2, \ldots, q_{ne})$ and $\boldsymbol{d}=(d_1, d_2, \ldots,d_m)$ are the force density vector and element diameter vector, respectively, and *ne* represents the number of elements in the frame structure and *m* represents the number of elements which are allowed to change their diameter; *C* is the compliance of the frame structure, calculated by the stiffness matrix $\boldsymbol{K}$ and displacement vector $\boldsymbol{U}$ of the frame structure; $V_{initial}$ is the initial volume the frame structure and $\boldsymbol{q}_{lower}$, $\boldsymbol{q}_{upper}$, $\boldsymbol{d}_{lower}$, $\boldsymbol{d}_{upper}$, $\boldsymbol{x}_{lower}$, $\boldsymbol{x}_{upper}$, $\boldsymbol{y}_{lower}$ and $\boldsymbol{y}_{upper}$ are the lower and upper bounds for $\boldsymbol{q}$, $\boldsymbol{d}$, $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively. It is remarkable that bounds for $\boldsymbol{x}$ and $\boldsymbol{y}$ are not always necessary. Sequential quadratic programming(SQP) is implemented to solve this problem. Note that the force density here has no relation to the axial force of deformed element, and it is adopted as design variable for defining the structure shape.

A brief description of the optimization procedure is as follows. Firstly, after obtaining the finite element model of the frame structure, set the supporting nodes and loading nodes as fix nodes and other nodes as free nodes, and calculate its connection matrix. Then randomly choose an initial value for force density vector $\boldsymbol{q}=(q_1, q_2, \ldots, q_n)$ and $\boldsymbol{d}=(d_1, d_2, \ldots,d_m)$, where Secondly, according to the self-equilibrium equation, the nodal coordinates of free nodes can be calculated by

$$\boldsymbol{C}_{free}^{T}\boldsymbol{Q}\boldsymbol{C}_{free}\boldsymbol{x}_{free}=\boldsymbol{P}_{x}^{*}-\boldsymbol{C}_{free}^{T}\boldsymbol{Q}\boldsymbol{C}_{fix}\boldsymbol{x}_{fix},\boldsymbol{C}_{free}^{T}\boldsymbol{Q}\boldsymbol{C}_{free}\boldsymbol{y}_{free}=\boldsymbol{P}_{y}^{*}-\boldsymbol{C}_{free}^{T}\boldsymbol{Q}\boldsymbol{C}_{fix}\boldsymbol{y}_{fix} \tag{2}$$

where $\boldsymbol{x}_{free}$ and $\boldsymbol{y}_{free}$ are the coordinates of free nodes, and $\boldsymbol{x}_{fix}$ and $\boldsymbol{y}_{fix}$ are the coordinates of fix nodes; $\boldsymbol{C}_{free}$ and $\boldsymbol{C}_{fix}$ are the connection matrices with respect to free nodes and fix nodes; $\boldsymbol{Q}$ is the diagonal matrix of force density vector $\boldsymbol{q}$, and $\boldsymbol{p}_x^*$ and $\boldsymbol{p}_y^*$ are the nodal load vector corresponding to free nodes. Since the loading nodes are included in fix nodes, $\boldsymbol{p}_x^*$ and $\boldsymbol{p}_y^*$ will be a zero vector, details can be found in reference [1, 2]. Thirdly, based on the new location of free nodes and the value of element diameter vector $\boldsymbol{d}$, the new shape of frame structure is derived as well as its stiffness matrix $\boldsymbol{K}$. Thus, the displacement vector $\boldsymbol{U}$ can be calculated by solving the stiffness equation and the compliance of *frame* structure is obtained according to Eq. (1). Since SQP is a gradient-based method for nonlinear programming, the sensitivity coefficients of objective and constraints function are also computed for reducing computation time. Terminate the optimization process if the optimality is satisfied; otherwise, modify the design variables $\boldsymbol{d}$ and $\boldsymbol{q}$ and repeat the optimization procedure. Finally, when the optimal diameter and nodal locations are obtained, change the topology of the frame structure by removing elements whose diameter reaches the lower bound d_{lower} and further optimize the diameter and nodal locations by using a traditional optimization method of SQP. Numerical examples are investigated to testified the accuracy and efficiency of the proposed method.

Reference

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