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Multiobjective Robust Shape and Topology Optimization of Plane Frames using Order Statistics

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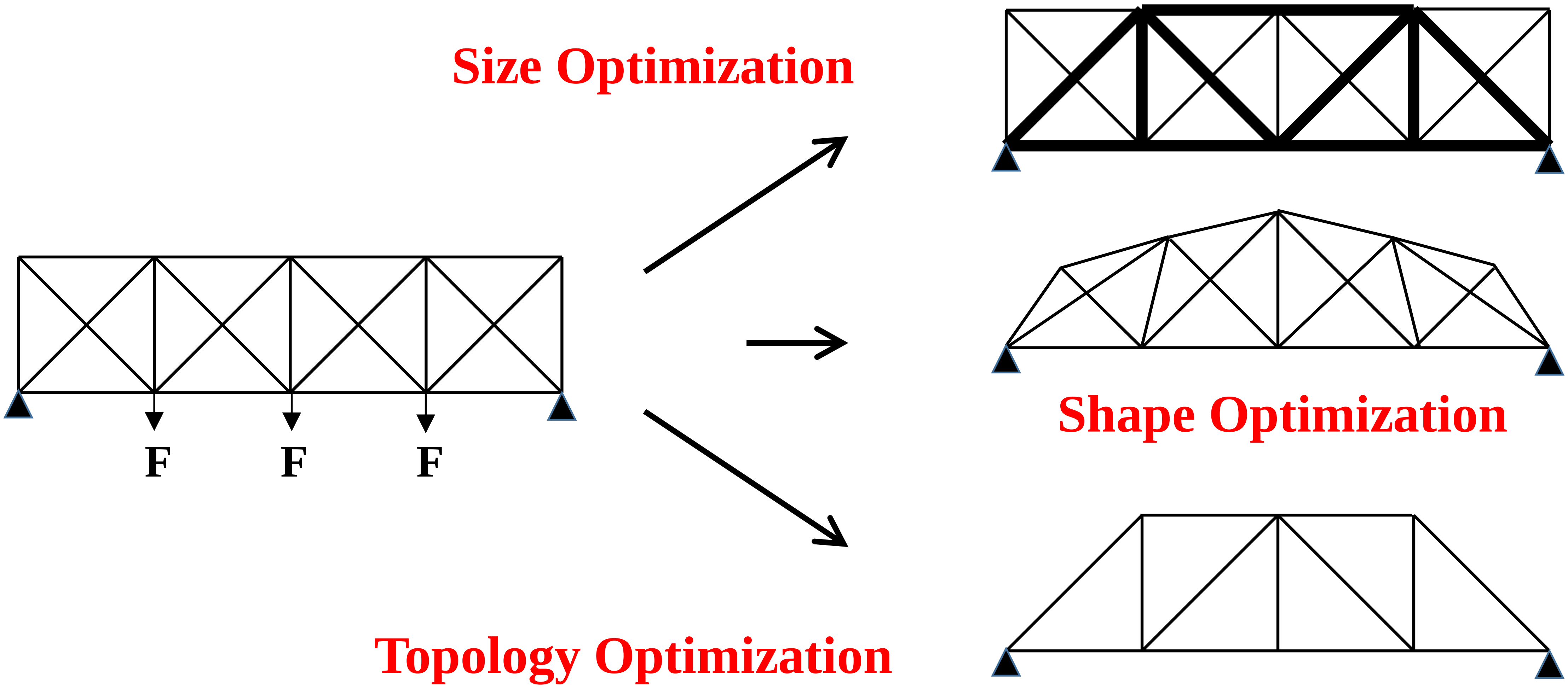
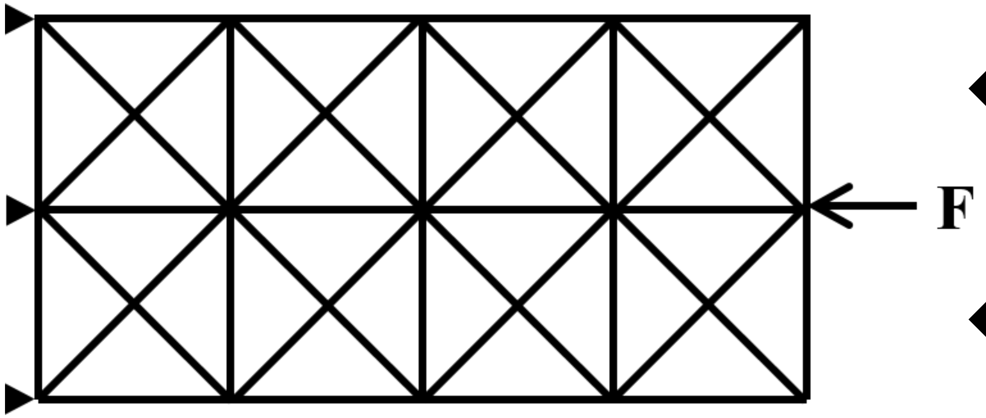
Background


Size Optimization
F
F
F
Shape Optimization
Topology Optimization

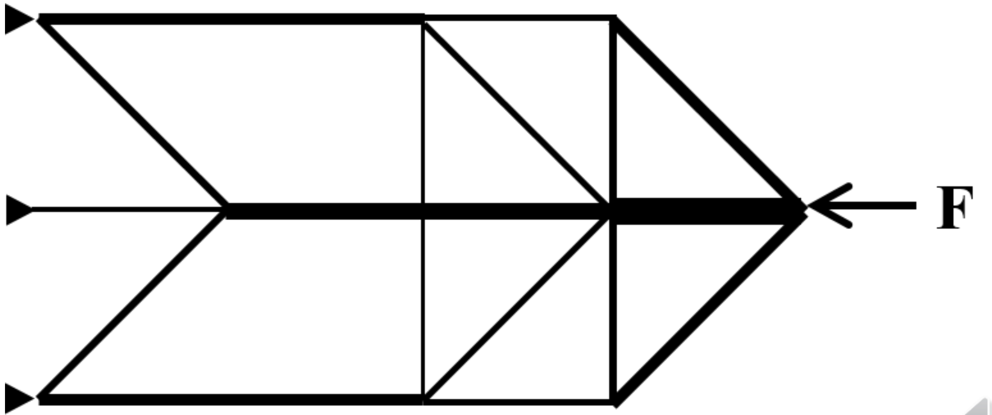
Background

Uncertainty in real world



Deterministic design

Considering Uncertainty



Suppose uncertainty exist in Young’s modulus

Source: A. Asadpoure *et al*, Robust topology optimization of structures with uncertainties in stiffness - Application to truss structures, 2011, Computers and Structures.

Existing Challenges

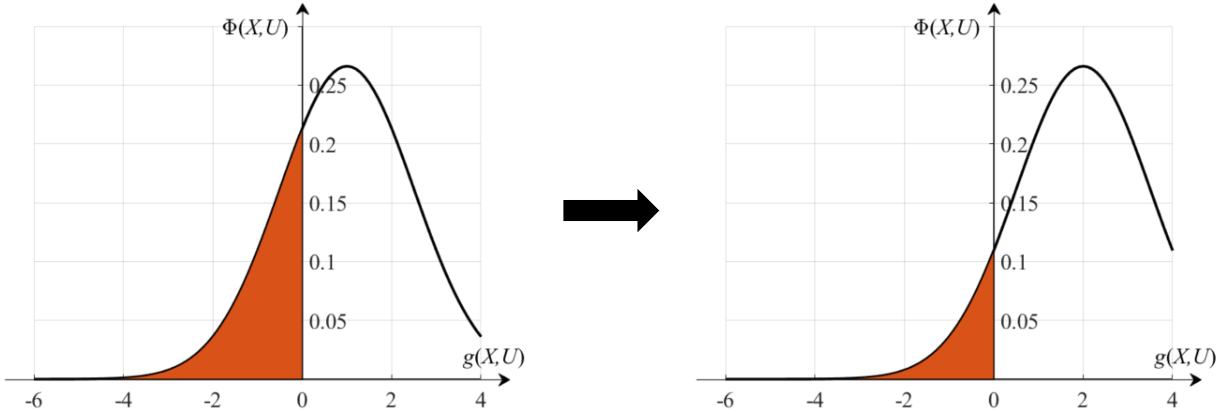
- **(1) Unknown distribution of uncertainty**

Two kinds of probability-based design

(i) Reliability-based design

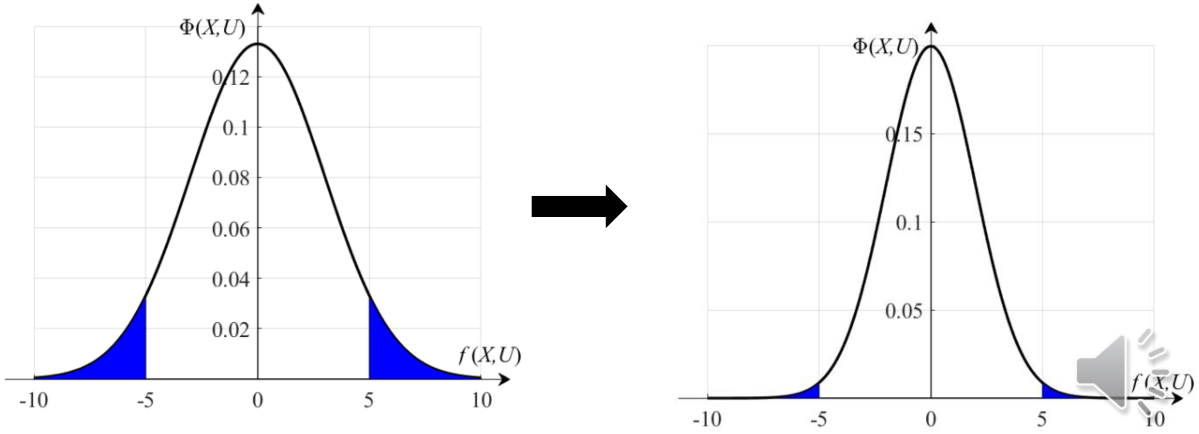
Minimize: $f\left(\boldsymbol{X};\boldsymbol{U}\right)$

Subject to: $P_{\text{fail}} = \int_{g\left(\boldsymbol{X};\boldsymbol{U}\right)<0} \Phi\left(\boldsymbol{X};\boldsymbol{U}\right) d\boldsymbol{U} \le \bar{P}$



(ii) Robust design

Minimize: $E\left(f\left(\boldsymbol{X};\boldsymbol{U}\right)\right) + k\sigma\left(f\left(\boldsymbol{X};\boldsymbol{U}\right)\right)$

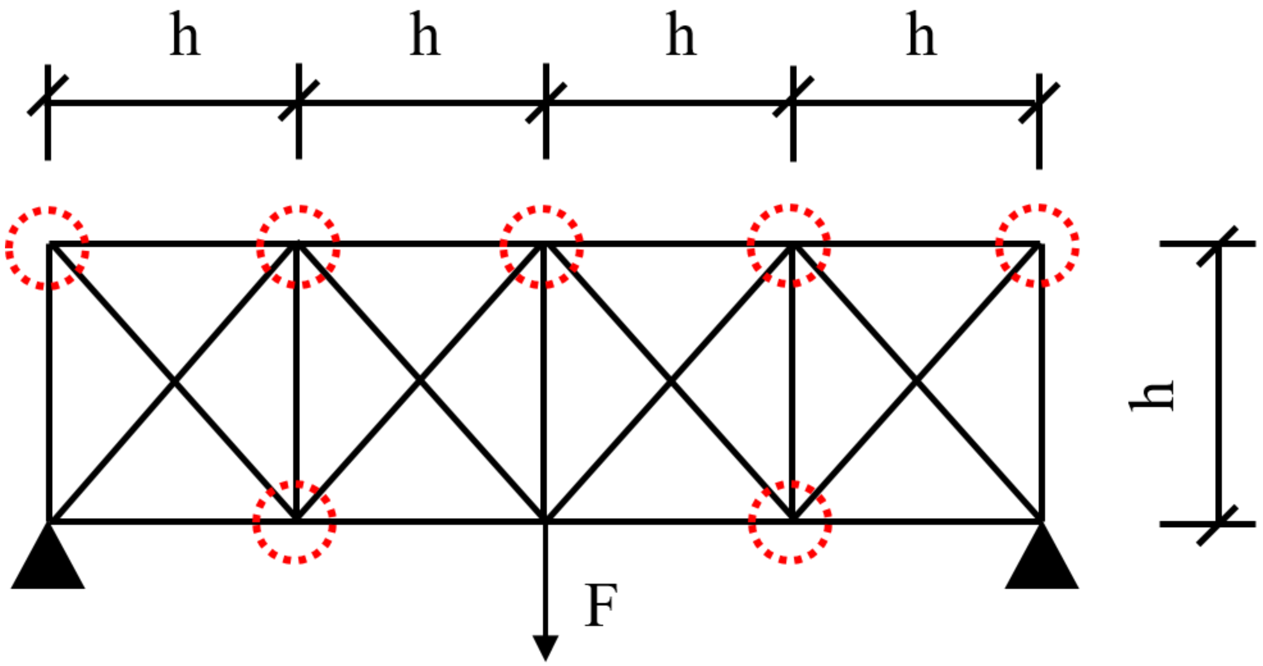


Existing Challenges

- **(2) Exact value of worst-case event**

Handle the uncertainty with worst-case design

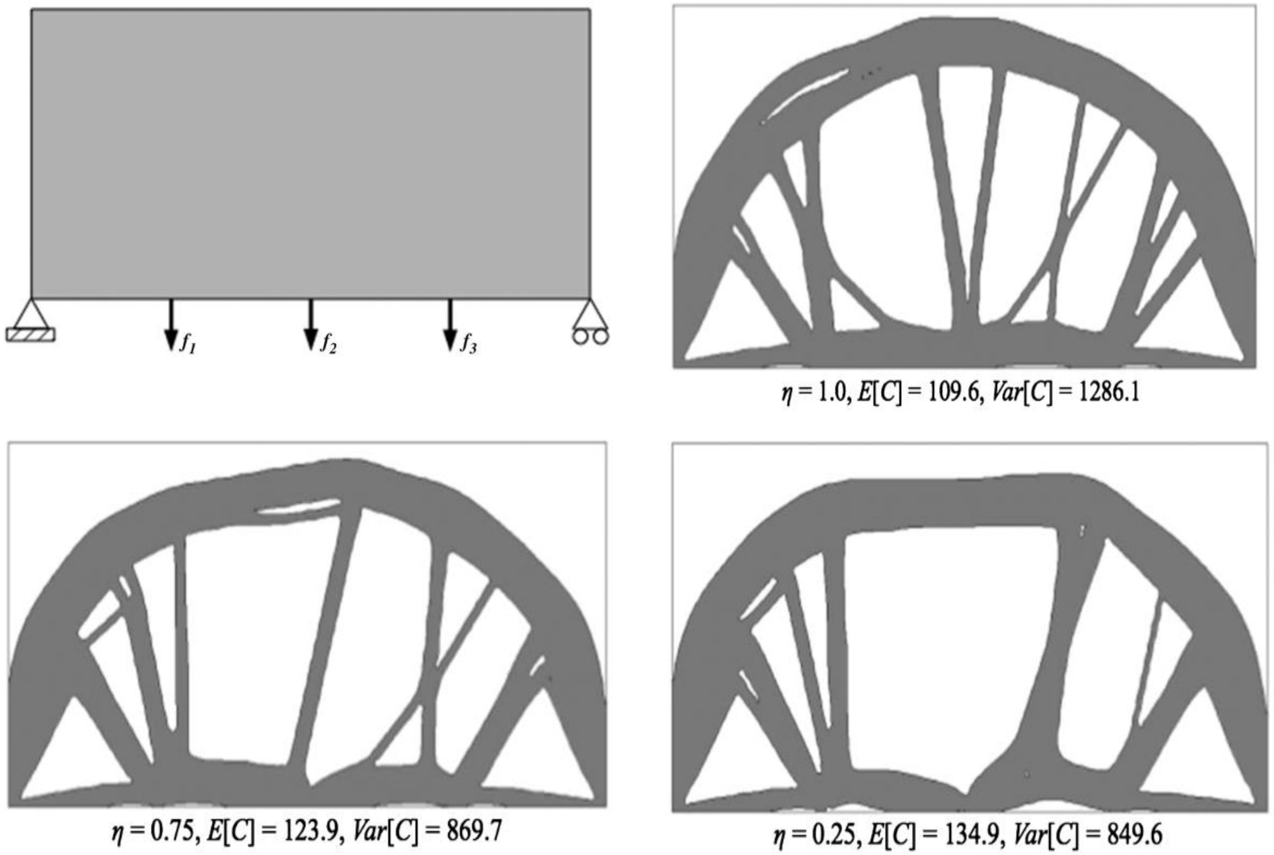
$$\text{Minimize: } f^{\max}\left(\boldsymbol{X};\boldsymbol{U}\right)=\max_{\boldsymbol{U}\in\Theta} f\left(\boldsymbol{X};\boldsymbol{U}\right)$$



Hard to obtain the exact worst structural response even if Θ is simple

Existing Challenges

- **(3) Trade-off relationships**



Robustness level

Trade-off

Nominal structural response

To what extend is the structure robust?

Source: P. Dunning *et al*, Robust Topology Optimization: Minimization of Expected and Variance of Compliance, 2013, AIAA Journal

Proposed Method

- **(1) Problem statement**

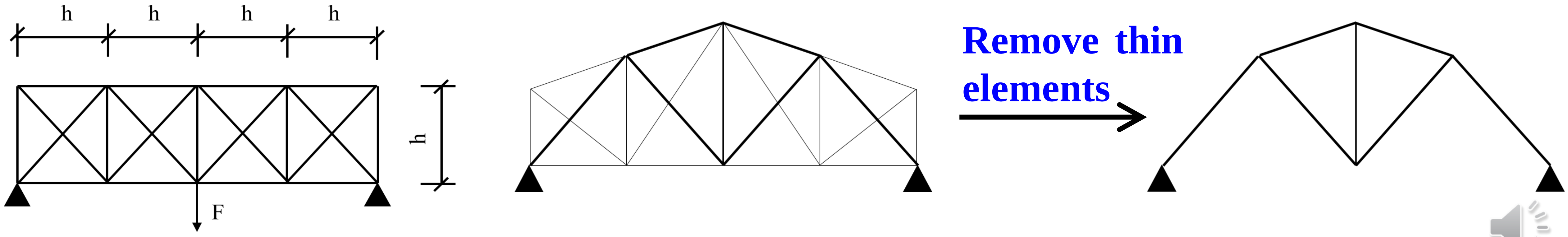
First consider a deterministic optimization problem

$$\text{Minimize}\quad \sigma(\boldsymbol{A},\boldsymbol{x}) = \max_{i=1,2,\ldots,n_{\mathrm{e}}} \sigma_{\max\text{-}i}(\boldsymbol{A},\boldsymbol{x})$$

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_{nx}, y_1, y_2, \ldots, y_{ny})$$

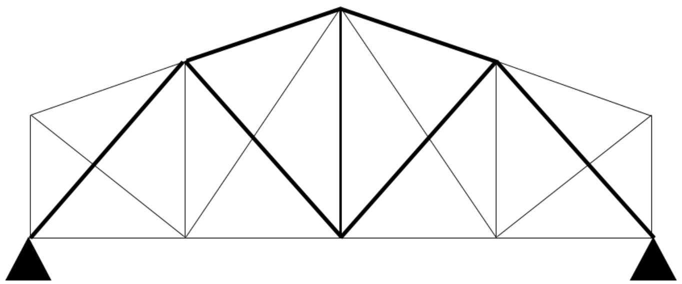
$$\text{subject to } V(\boldsymbol{A},\boldsymbol{x}) \le V_{\mathrm{U}};\ x_{\mathrm{L}} \le x_{j_1} \le x_{\mathrm{U}},\ j_1 = 1,2,\ldots,n_x;$$

$$y_{\mathrm{L}} \le y_{j_2} \le y_{\mathrm{U}},\ j_2 = 1,2,\ldots,n_y;\ A_{\mathrm{L}} \le A_l \le A_{\mathrm{U}},\ l = 1,2,\ldots,n_{\mathrm{e}}$$

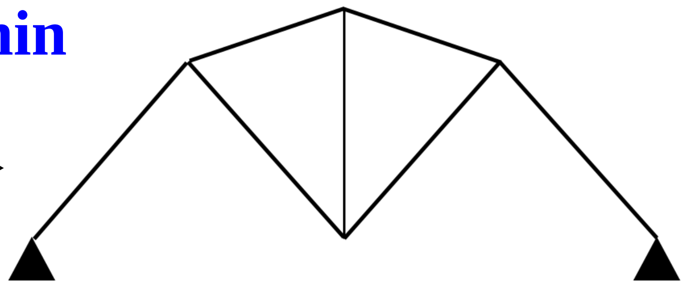


Proposed Method

- **(1) Problem statement**



Remove thin elements



Some of the thin elements is helpful to resist uncertainty

Second consider a robust optimization with design variables *A*

$$\text{Minimize} \quad \sigma^{\max}\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)=\max_{\boldsymbol{E}_{\mathrm{p}}\in\Theta_{E_{\mathrm{p}}},\boldsymbol{x}_{\mathrm{p}}\in\Theta_{\boldsymbol{x}_{\mathrm{p}}}}\left\{\max_{i=1,2,\ldots n_e}\sigma_{\max\text{-}i}\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)\right\}$$

$$\text{subject to } V\left(\boldsymbol{A}\right)\le V_{\mathrm{U}};\ A_{\mathrm{L}}\le A_l\le A_{\mathrm{U}},\quad l=1,2,\ldots,n_{\mathrm{e}}$$

Uncertainty in *E* and *x*

Proposed Method

- **(2) Approximate worst-case using order statistics**

$$\text{Minimize } \boxed{\sigma^{\max}\left(\boldsymbol{A}; \boldsymbol{E}_{\mathrm{p}}, \boldsymbol{x}_{\mathrm{p}}\right)} = \max_{\boldsymbol{E}_{\mathrm{p}} \in \Theta_{E_{\mathrm{p}}}, \boldsymbol{x}_{\mathrm{p}} \in \Theta_{x_{\mathrm{p}}}} \left\{ \max_{i=1,2,\ldots n_e} \sigma_{\max\text{-}i}\left(\boldsymbol{A}; \boldsymbol{E}_{\mathrm{p}}, \boldsymbol{x}_{\mathrm{p}}\right) \right\}$$

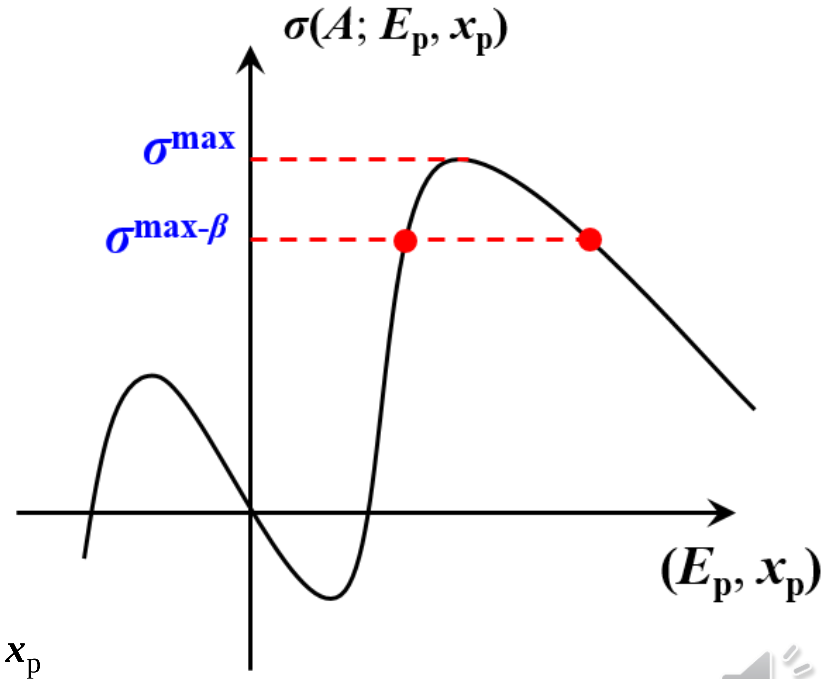
$$\text{subject to } V\left(\boldsymbol{A}\right) \le V_{\mathrm{U}};\ A_{\mathrm{L}} \le A_l \le A_{\mathrm{U}},\quad l = 1, 2, \ldots, n_{\mathrm{e}}$$

Relaxed

100*β*th (0<*β*<1) quantile of structural stress $\sigma^{\max\text{-}\beta}$

$$\text{Probability}\left\{\sigma\left(\boldsymbol{A}; \boldsymbol{E}_{\mathrm{p}}, \boldsymbol{x}_{\mathrm{p}}\right) \le \sigma^{\max-\beta}\right\} = \beta$$

$$\sigma\left(\boldsymbol{A}; \boldsymbol{E}_{\mathrm{p}}, \boldsymbol{x}_{\mathrm{p}}\right) = \max_{i=1,2,\ldots n_e} \sigma_{\max\text{-}i}\left(\boldsymbol{A}; \boldsymbol{E}_{\mathrm{p}}, \boldsymbol{x}_{\mathrm{p}}\right), \forall \boldsymbol{E}_{\mathrm{p}} \in \Theta_{E_{\mathrm{p}}}, \forall \boldsymbol{x}_{\mathrm{p}} \in \Theta_{x_{\mathrm{p}}}$$

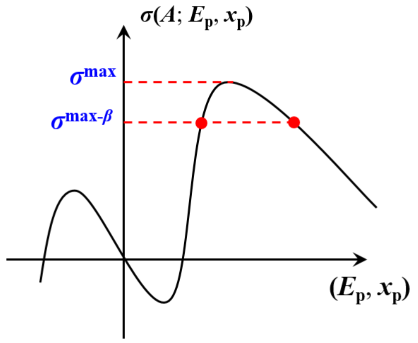


Proposed Method

- **(2) Approximate worst-case using order statistics**

$$\text{Probability}\left\{\sigma\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)\leq\sigma^{\max-\beta}\right\}=\beta$$

$$\sigma\left(\boldsymbol{A};\boxed{\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}}\right)=\max_{i=1,2,\ldots n_e}\sigma_{\max\text{-}i}\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right),\forall\boldsymbol{E}_{\mathrm{p}}\in\Theta_{E_{\mathrm{p}}},\forall\boldsymbol{x}_{\mathrm{p}}\in\Theta_{\boldsymbol{x}_{\mathrm{p}}}$$



Given m sets of $(E_p, x_p)_1$, …, $(E_p, x_p)_m$, we can obtain the following

$$\sigma_1^{\max}=\sigma\left(\boldsymbol{A};\left(\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)_1\right),\sigma_2^{\max}=\sigma\left(\boldsymbol{A};\left(\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)_2\right),\cdots,\sigma_m^{\max}=\sigma\left(\boldsymbol{A};\left(\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)_m\right)$$

and place them in a descending order

$$\sigma_{1:m}^{\max}\geq\ldots\geq\sigma_{k:m}^{\max}\geq\ldots\geq\sigma_{m:m}^{\max},1\leq k\leq m$$

Proposed Method

- **(2) Approximate worst-case using order statistics**

Based on the statistical inference theory of order statistics

$$\alpha_k = \Pr\left\{\boxed{\Pr\left\{\sigma\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right) \le \sigma_{k:m}^{\max}\right\}} \ge \beta\right\} = \sum_{r=0}^{m-k} \binom{m}{r} \beta^r (1-\beta)^{m-r}$$

$\boldsymbol{\alpha_k \to 1,\ \sigma_{k:m}^{\max} \to \sigma^{\max\text{-}\beta}}$

Given α_k and sample size m, we can obtain the relation between k and β

Relation between k and β ($\alpha_k = 0.9$, $m = 200$)

k	1	2	3	4	5	6	7	8	9	10
β	0.989	0.981	0.974	0.967	0.960	0.954	0.948	0.942	0.936	0.930
k	11	12	13	14	15	16	17	18	19	20
β	0.924	0.918	0.912	0.907	0.901	0.895	0.890	0.884	0.878	0.873

Proposed Method

- **(2) Approximate worst-case using order statistics**

100βth (0<β<1) quantile structural stress $\sigma^{\text{max-}\beta}$

$$\Pr\left\{\sigma\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)\le \boxed{\sigma^{\max-\beta}}\right\}=\beta$$

$$\sigma\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)=\max_{i=1,2,\ldots,n_e}\sigma_{\max\text{-}i}\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right),\forall\boldsymbol{E}_{\mathrm{p}}\in\Theta_{E_{\mathrm{p}}},\forall\boldsymbol{x}_{\mathrm{p}}\in\Theta_{\boldsymbol{x}_{\mathrm{p}}}$$

Approximated

***k*th order statistics $\sigma_{k:m}^{\max}$ with sample size *m* and confidence level α_k**

Structural robustness $\xrightarrow{\textbf{Represented}}$ **Order *k* and $\sigma_{k:m}^{\max}$**

Proposed Method

- **(3) Approximate worst-case design using order statistics**

$$\text{Minimize} \quad \sigma^{\max}\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)=\max_{\boldsymbol{E}_{\mathrm{p}}\in\Theta_{E_{\mathrm{p}}},\boldsymbol{x}_{\mathrm{p}}\in\Theta_{x_{\mathrm{p}}}}\left\{\max_{i=1,2,\ldots n_e}\sigma_{\max\text{-}i}\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)\right\}$$

$$\text{subject to } V\left(\boldsymbol{A}\right)\le V_{\mathrm{U}};\ A_{\mathrm{L}}\le A_l\le A_{\mathrm{U}},\quad l=1,2,\ldots,n_{\mathrm{e}}$$

is rewritten by using order statistics

$$\text{Minimize} \quad \sigma_{k:m}^{\max}\left(\boldsymbol{A};\boldsymbol{E}_{\mathrm{p}},\boldsymbol{x}_{\mathrm{p}}\right)$$

$$\text{subject to } V\left(\boldsymbol{A}\right)\le V_{\mathrm{U}};\ A_{\mathrm{L}}\le A_l\le A_{\mathrm{U}},\quad l=1,2,\ldots,n_{\mathrm{e}}$$

Smaller order k → Larger robustness level

Proposed Method

- **(3) Multiobjective optimization problem**

To minimize structural stress at different robustness levels

$$\text{Minimize} \quad \bar{\sigma}^{\max}, \sigma_{k_1:m}^{\max}, \sigma_{k_2 m}^{\max}, \ldots, \sigma_{k_3:m}^{\max}$$

$$\text{subject to } V(\boldsymbol{A}) \le V_{\mathrm{U}};\ A_{\mathrm{L}} \le A_l \le A_{\mathrm{U}},\quad l = 1, 2, \ldots, n_{\mathrm{e}}$$

and $\bar{\sigma}\left(\boldsymbol{A}; \bar{\boldsymbol{E}}_{\mathrm{p}}, \bar{\boldsymbol{x}}_{\mathrm{p}}\right) = \max\limits_{i=1,2,\ldots n_e} \sigma_{\max\text{-}i}\left(\boldsymbol{A}; \bar{\boldsymbol{E}}_{\mathrm{p}}, \bar{\boldsymbol{x}}_{\mathrm{p}}\right)$ **with nominal values of uncertainties**

In this study, *m* = 200 and *k* = 1, 50, and 100, respectively

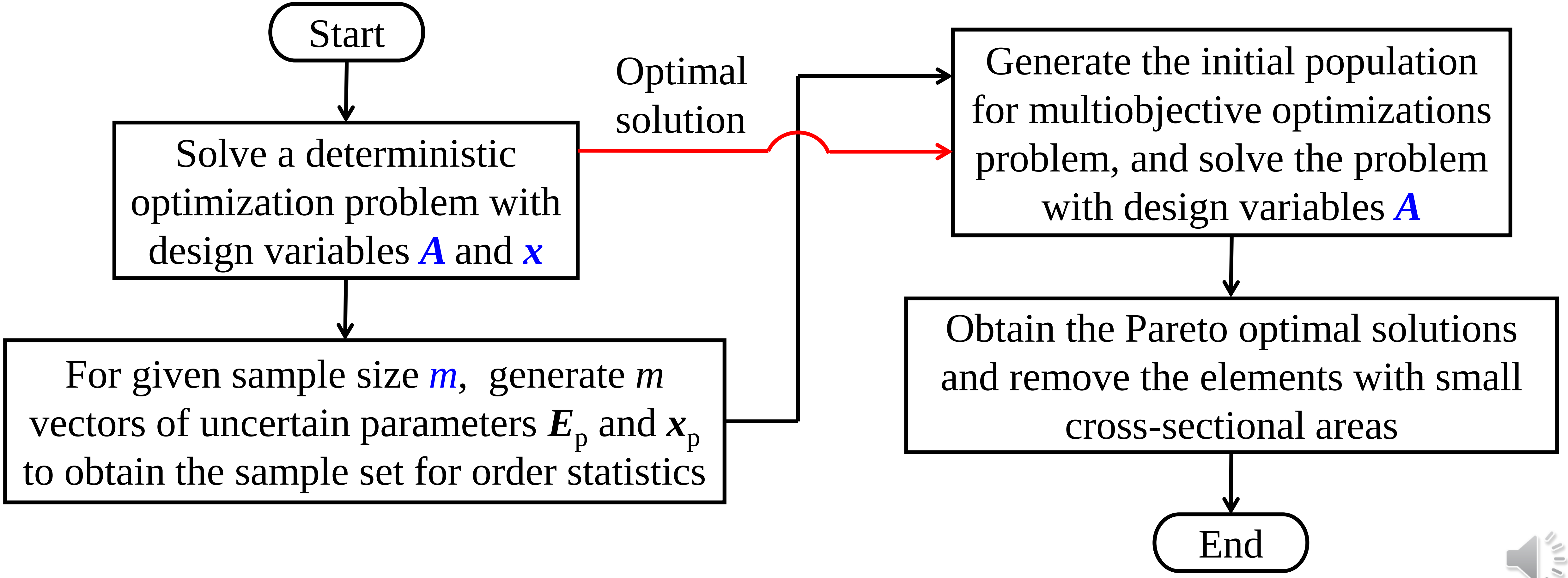
$$\text{Minimize} \quad \bar{\sigma}^{\max}, \sigma_{1:200}^{\max}, \sigma_{50:200}^{\max}, \sigma_{100:200}^{\max}$$

$$\text{subject to } V(\boldsymbol{A}) \le V_{\mathrm{U}};\ A_{\mathrm{L}} \le A_l \le A_{\mathrm{U}},\quad l = 1, 2, \ldots, n_{\mathrm{e}}$$

Solved by genetic algorithms (GA)

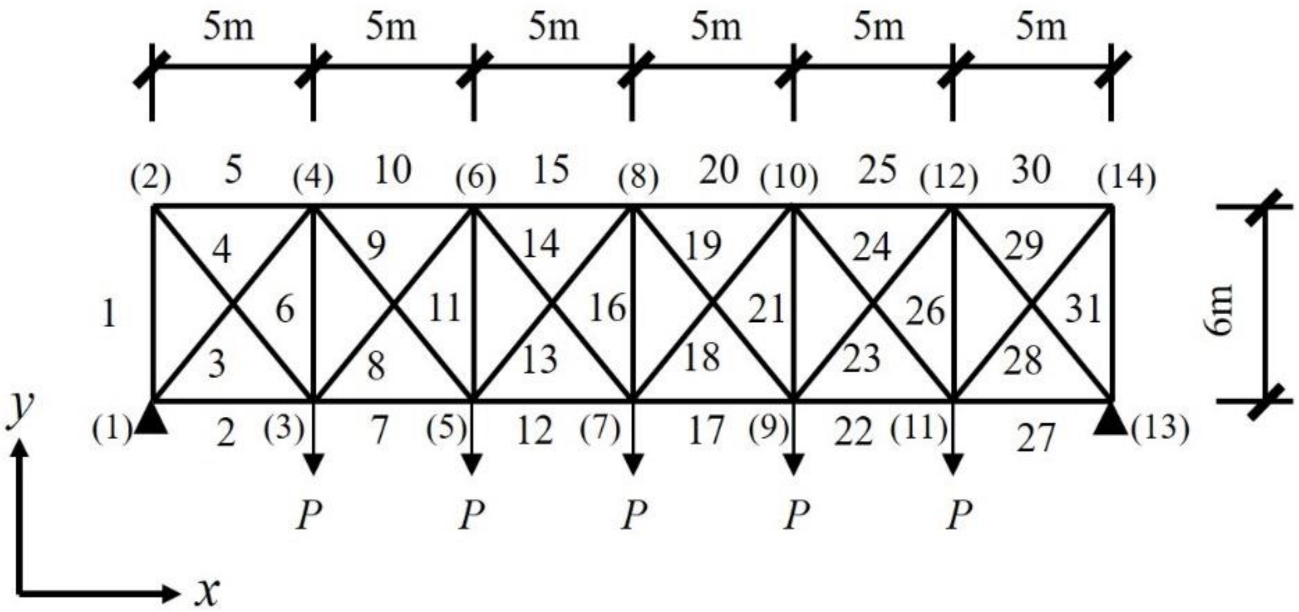
Proposed Method

- **(4) Flowchart of the optimization procedure**



Numerical Example

- **Example 1:**

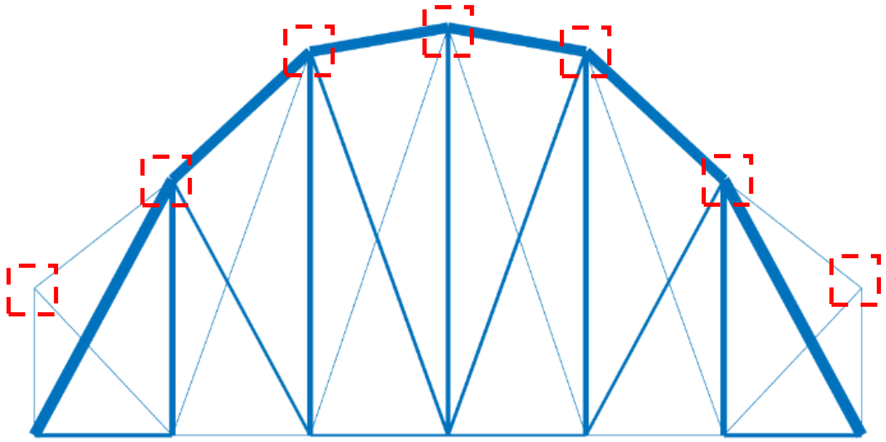


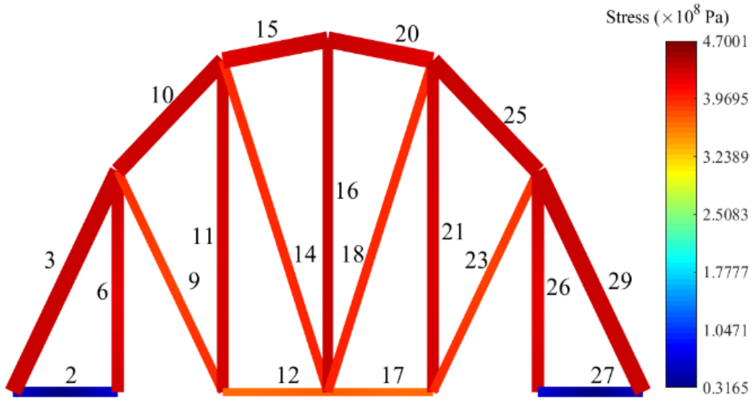
Initial condition:
P = 3×10^6 N; E = 2.1×10^{11} Pa;
V_U = 1m^3

Design variables (deterministic):
$\boldsymbol{A} = (A_1, A_2, \ldots, A_{16})$;
$\boldsymbol{x} = (y_2, y_4, y_6, y_8)$

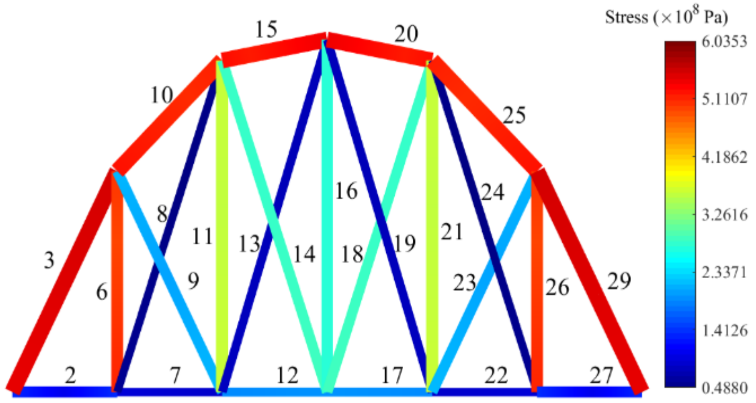
Numerical Example

➢ Example 1:

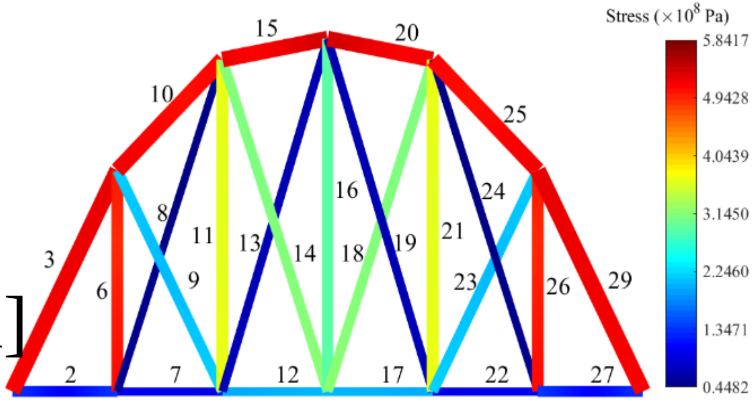




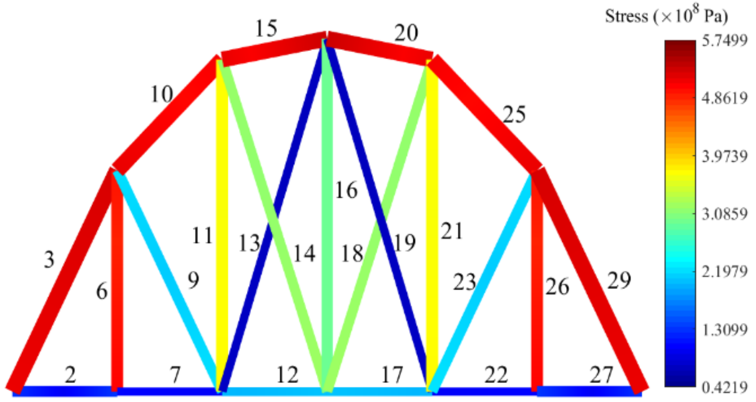
Solution A ($\bar{\sigma}^{\max}$)



Solution B ($\sigma^{\max}_{1:200}$)



Solution C ($\sigma^{\max}_{50:200}$)



Solution D ($\sigma^{\max}_{100:200}$)

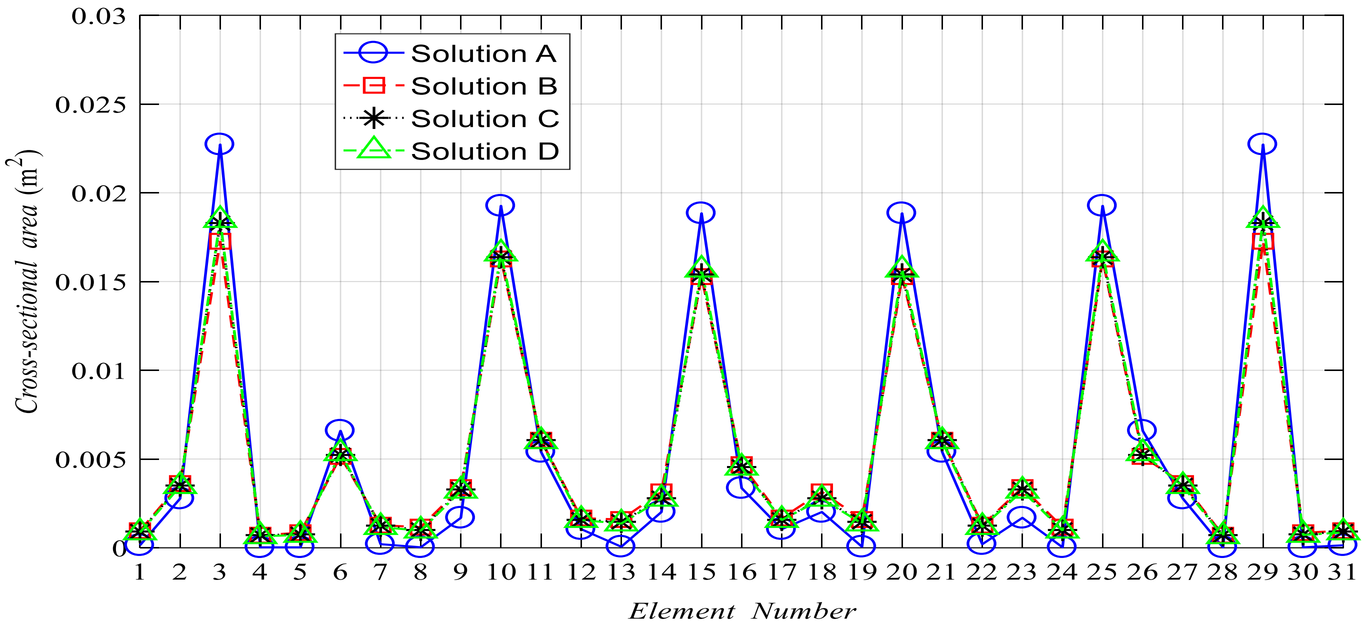
Uncertainty:

$\boldsymbol{E}_{\mathbf{p}} \in [1.89\times10^{11}\text{Pa},\ 2.31\times10^{11}\ \text{Pa}]$

$\boldsymbol{x}_{\mathbf{p}} \in [0.6\text{m},\ 0.6\text{m}]$

Numerical Example

- **Example 1:**



Solution	$\overline{\sigma}^{max}$ (Pa)	$\sigma_{1:200}^{max}$ (Pa)	$\sigma_{50:200}^{max}$ (Pa)	$\sigma_{100:200}^{max}$ (Pa)	Volume (m^3)
A	4.7001×10^8	5.9733×10^9	2.8549×10^9	2.0861×10^9	1.0
B	6.0353×10^8	7.7531×10^8	6.9560×10^8	6.5546×10^8	1.0
C	5.8417×10^8	8.2934×10^8	6.8127×10^8	6.4418×10^8	1.0
D	5.7499×10^8	8.7466×10^8	6.9079×10^8	6.3814×10^8	1.0

Conclusions and future directions

➢ **The proposed method has the following conclusions:**

- **The worst value is approximated by the order statistics with specified confidence level, and the robustness level is represented by the order *k* and the corresponding order statistics regardless of the distribution of uncertainty.**
- **A multiobjective optimization problem is formulated with the choice of different robustness level.**
- **Topology of the structure may vary depending on the robustness level.**

Conclusions and future directions

- **Future directions：**

  - **Since GA is selected as the solver to the optimization problem, it will require large number of FEA and is not efficient when the time for one run of FEM is long, and a surrogate mode would be needed to alleviate such difficulty.**
  - **The optimization procedure includes two steps and might be complicated, however, it is more convenient to simultaneously optimize the shape and topology of the structure under uncertainty.**

Thanks for your kind attention